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Combined Laminar Forced and Free Convection Heat Transfer to Viscoelastic Fluids

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A correlating equation for combined laminar forced and free convection heat transfer to Newtonian fluids was proposed by Churchill (1977) and supported by Ruckenstein (1978). It was further shown by Shenoy (1980) that the same equation could be used for non-Newtonian power-law fluids except for the new definitions of Nusselt numbers for pure forced and free convection respectively. There exists no theoretical analysis of combined laminar forced and free convection heat transfer to viscoelastic fluids and hence the purpose of this communication is to study this problem, providing a sequel to the above efforts. Essentially, the idea is to use the same correlating equation as Churchill (1977), with renewed definitions of Nusselt numbers pertinent to viscoelastic fluids for pure forced and pure free convection respectively.

A theoretical analysis of laminar forced convection heat transfer to viscoelastic fluids is done by the approximate integral method. A similarity solution is obtained and is seen to exist only for the case of a second order fluid in the stagnation region of a constant temperature heated horizontal cylinder. Under exactly the same conditions, Shenoy and Mashelkar (1978) analyzed the laminar natural convection heat transfer to viscoelastic fluids. Using the two informations, an idea of the combined laminar forced and free convection heat transfer to viscoelastic fluids is obtained.

THEORETICAL ANALYSIS

For two-dimensional steady laminar forced convection flow of a viscoelastic fluid over an object indicated in Figure 1, the simplified non-dimensionalised boundary equations of momentum and energy in their integral forms can be written as

$$\frac{\partial}{\partial x_1}\int_0^{\delta_1}(u_1^2 - U_1^2)dy_1 = -\left(\frac{\partial u_1}{\partial y_1}\right)^n_{y_1=0} + Wi\,\frac{\partial}{\partial x_1}\int_0^{\delta_1}\left(\frac{\partial u_1}{\partial y_1}\right)^s dy_1 \quad (1)$$

$$\frac{\partial}{\partial x_1}\int_0^{\delta_{T_1}} u_1\theta dy_1 = -\frac{1}{Pr}\left(\frac{\partial \theta}{\partial y_1}\right)_{y_1=0} \quad (2)$$

where

$$x_1 = \frac{x}{l_c};\quad y_1 = \frac{y}{l_c};\quad u_1 = \frac{u}{U_\infty}$$

$$U_1 = \frac{U}{U_\infty};\quad \delta_1 = \frac{\delta}{l_c};\quad \delta_{T_1} = \frac{\delta_T}{l_c} \quad (3)$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}$$

and

$$Wi = \frac{2m}{\rho}\,\frac{U_\infty^{s-2}}{l_c^s} \quad (4)$$

$$Pr = \frac{\rho C_p}{k}\left(\frac{K}{\rho}\right)^{\frac{1}{n}} U_\infty^{\frac{2(n-1)}{n}} \quad (5)$$

The choice of the constitutive equation is the same as that of Shenoy and Mashelkar (1978) in their analysis of laminar natural convection heat transfer to a viscoelastic fluid and the above Equations (1) and (2) are generated by similar arguments. The boundary conditions on the velocity and temperature profiles are

$$u_1(x_1,0) = 0 \quad u_1(x_1,\delta_1) = 0$$
$$\theta(x_1,0) = 0 \quad \theta(x_1,\delta_{T_1}) = 0 \quad (6)$$

In line with the general tradition of an integral solution, the

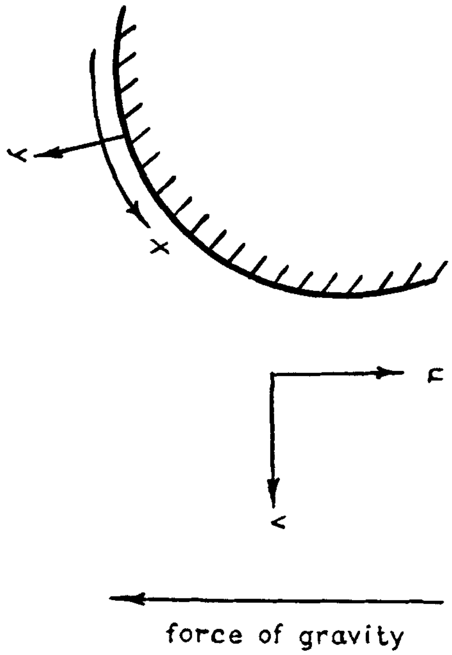

Figure 1. Schematic diagram of flow past a curved surface.

velocity profile (u_1) and the temperature profile (θ) are specified as

$$u_1 = U_1(2\eta - 2\eta^3 + \eta^4) \tag{7}$$

$$\theta = (1 + \eta_T)(1 - \eta_T)^3 \tag{8}$$

where

$$\eta = \frac{y_1}{\delta_1} \quad \text{and} \quad \eta_T = \frac{y_1}{\delta_{T_1}} \tag{9}$$

Equations (1) and (2) are now solved using Equations (7) and (8) to give

$$-\frac{3}{10}\frac{\partial}{\partial x_1}(\delta_1 U_1^2) = -\frac{2^n U_1^n}{\delta_1^n} + I\, Wi \frac{\partial}{\partial x_1}\left(\frac{U_1^s}{\delta_1^{s-1}}\right) \tag{10}$$

$$\frac{\partial}{\partial x_1}\left[\frac{\delta_{T_1}^2}{\delta_1} U_1 f_1(\alpha_F)\right] = \frac{2}{\delta_{T_1} Pr} \tag{11}$$

where

$$\alpha_F = \frac{\delta_T}{\delta} \tag{12}$$

$$f_1(\alpha_F) = \frac{2}{15} - \frac{3}{140}\alpha_F^2 + \frac{1}{180}\alpha_F^3 \tag{13}$$

$$I = \int_0^1 (2 - 6\eta^2 + 4\eta^3)^s\, d\eta \tag{14}$$

Assuming

$$\delta_{T_1} = B_1 x_1^r \tag{15}$$

$$\delta_1 = B_2 x_1^t \tag{16}$$

$$U_1 = B_3 x_1^p \tag{17}$$

a similarity search is carried out to give

$$-\frac{3}{5} B_2 B_3^2 p x_1^{2p+t-1} = -\frac{2^n B_3^n x_1^{n(p-t)}}{B_2^n} + I\, Wi[sp - (s-1)t]\frac{B_3^s x_1^{sp-(s-1)t-1}}{B_2^{s-1}} \tag{18}$$

and

$$\frac{B_1^2 B_3}{B_2}\frac{\partial}{\partial x_1}\left\{x_1^{2r+p-t}\left[\frac{2}{15} - \frac{3}{140}\left(\frac{B_1}{B_2}\right)^2 x_1^{2(r-t)} + \frac{1}{180}\left(\frac{B_1}{B_2}\right)^3 x_1^{3(r-t)}\right]\right\} = \frac{2x_1^{-r}}{B_1 Pr} \tag{19}$$

For similarity to exist it readily follows that $r = t$ and furthermore that

$$2p + t - 1 = n(p - t) = sp - (s-1)t - 1 \tag{20}$$

and

$$2r + p - t - 1 = -r \tag{21}$$

yielding

$$r = \frac{1 + (n-2)p}{n+1} = \frac{1-p}{2} \tag{22}$$

and

$$s = n + \frac{1-r}{p-r} = \frac{2p}{p-r} \tag{23}$$

Equations (22) and (23) are simultaneously satisfied for $s = 2$, $n = 1$, $p = 1$ and $r = t = 0$, which is the same realistic condition obtained by Shenoy and Mashelkar (1978) for laminar natural convection. This similarity solution is for the case of a second order fluid in the stagnation region of a constant temperature heated horizontal cylinder. For a horizontal cylinder a reasonable potential flow as given in Schlichting (1968) can be used

$$U_1 = 2\sin x_1 \approx 2x_1 \text{ (for small } x_1\text{)} \tag{24}$$

Thus $B_3 = 2$ and the simplified forms of Equations (18) and (19) can be written as

$$\frac{3}{5}B_2 = \frac{1}{B_2} - \frac{104}{35}\frac{Wi}{B_2} \tag{25}$$

$$\frac{B_1^2}{B_2} f_1(\alpha_F) = \frac{1}{B_1\, Pr} \tag{26}$$

Solving the above gives

$$B_2 = \left[\frac{5}{3}\left(1 - \frac{104}{35}Wi\right)\right]^{\frac{1}{2}} \tag{27}$$

$$B_1 = \frac{\left[\frac{5}{3}\left(1 - \frac{104}{35}Wi\right)\right]^{\frac{1}{6}}}{[f_1(\alpha_F)]^{\frac{1}{3}}\, Pr^{\frac{1}{3}}} \tag{28}$$

The relationship between α_F & Wi can be obtained from

$$\frac{35}{104}\left[1 - \frac{3}{5\alpha_F^3 f_1(\alpha_F) Pr}\right] = Wi \tag{29}$$

Now the local Nusselt number is defined as

$$Nu_{x,F} = \frac{\left(-\frac{\partial T}{\partial y}\right)_{y=0} x}{\Delta T} \tag{30}$$

$$= \frac{2}{B_1} Re_x^{\frac{1}{2}} x_1^{\frac{p}{2}} \tag{31}$$

For the stagnation region of a constant temperature heated horizontal cylinder, $p = 1$ and $l_c = R$ (radius of the cylinder), thus giving

$$Nu_{x,F} = \frac{2[f_1(\alpha_F)]^{\frac{1}{3}}}{\left[\frac{5}{3}\left(1 - \frac{104}{35}Wi\right)\right]^{\frac{1}{6}}} Re_x^{\frac{1}{2}}\, Pr^{\frac{1}{3}}\left(\frac{x}{R}\right)^{\frac{1}{2}} \tag{32}$$

where

$$Wi = \frac{2m}{\rho R^2}; \quad Pr = \frac{\mu C_p}{k}; \quad Re_x = \frac{\rho U_\infty x}{\mu} \tag{33}$$

The average Nusselt number can now be easily obtained as

$$Nu_{avR,F} = \frac{2[f_1(\alpha_F)]^{\frac{1}{3}}}{\left[\frac{5}{3}\left(1 - \frac{104}{35}Wi\right)\right]^{\frac{1}{6}}} Re_R^{\frac{1}{2}}\, Pr^{\frac{1}{3}} \tag{34}$$

To check the propriety of the above analysis, a comparison of Nusselt numbers obtained for $Wi = 0$ from the present work is done with the results of Squire (1938) for Newtonian fluids and good agreement is seen, from Table 1.

For pure laminar natural convection heat transfer to a second order fluid in the stagnation region of a constant temperature heated horizontal cylinder, Shenoy and Mashelkar (1978) show

TABLE 1. NUSSELT NUMBERS OBTAINED FOR $Wi = 0$ COMPARED WITH SQUIRE (1938) FOR NEWTONIAN FLUIDS.

Pr	7.00	10.0	15.0
$\frac{Nu_D}{2Re_D^{1/2}}$ (Squire)	1.18	1.34	1.54
$\frac{Nu_D}{2Re_D^{1/2}}$ (present work)	1.22	1.39	1.60

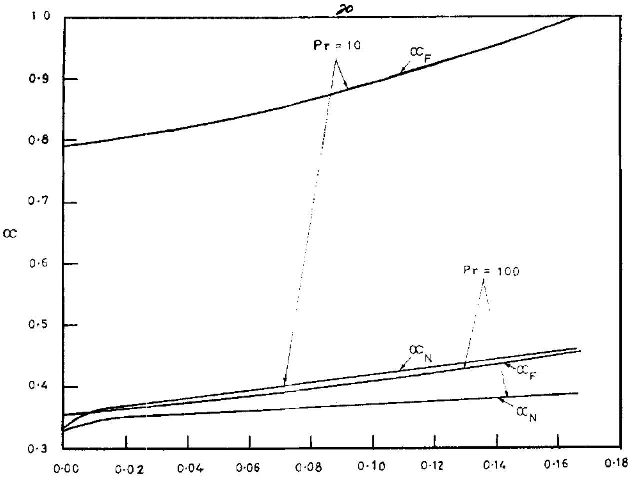

Figure 2. Variation of the ratio of boundary layer thicknesses with viscoelasticity.

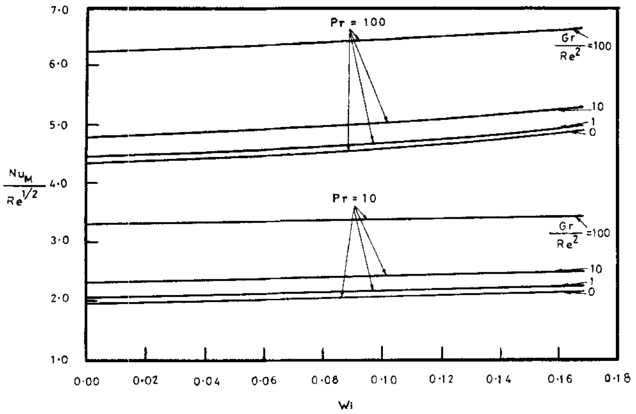

Figure 3. Variation of average Nusselt number for mixed convection with viscoelasticity.

that

$$Nu_{x,N} = 2\left[\frac{(297\alpha_N - 50)f_2(\alpha_N)}{980\alpha_N^2}\right]^{\frac{1}{4}} Gr_x^{\frac{1}{4}}\, Pr^{\frac{1}{4}}\left(\frac{x}{R}\right)^{\frac{1}{4}} \tag{35}$$

and

$$Nu_{avR,N} = 2\left[\frac{(297\alpha_N - 50)f_2(\alpha_N)}{980\alpha_N^2}\right]^{\frac{1}{4}} Gr_R^{\frac{1}{4}}\, Pr^{\frac{1}{4}} \tag{36}$$

where

$$f_2(\alpha_N) = \frac{1}{15}\alpha_N - \frac{5}{42}\alpha_N^2 + \frac{3}{28}\alpha_N^3 - \frac{1}{18}\alpha_N^4 + \frac{1}{63}\alpha_N^5 - \frac{3}{1540}\alpha_N^6 \tag{37}$$

$$Gr_x = \frac{\rho^2 x^3[g\beta(T_w - T_\infty)]}{\mu^2}; \quad Gr_R = \frac{\rho^2 R^3[g\beta(T_w - T_\infty)]}{\mu^2}; \quad Pr = \frac{\mu C_p}{k} \tag{38}$$

The relationship between α_N and Wi is given by

$$\left[\frac{245 f_2(\alpha_N)\, Pr}{(297\alpha_N - 50)}\right]^{\frac{1}{2}} \left[\frac{495\alpha_N(3\alpha_N - 1)}{(297\alpha_N - 50)}\right] = Wi \tag{39}$$

where

$$Wi = \frac{2m}{\rho R^2} \tag{40}$$

In a manner similar to Ruckenstein (1978) and Shenoy (1980), it can be easily shown that for combined forced and free convection heat transfer in the case considered herein the following equation can be used

$$Nu_{avR,M}^3 = Nu_{avR,F}^3 + Nu_{avR,N}^3 \tag{41}$$

thus

$$\frac{Nu_{avR,M}}{Re_R^{\frac{1}{2}}} = 2\left\{\frac{f_1(\alpha_F)\, Pr}{\left[\frac{5}{3}\left(1 - \frac{104}{35}Wi\right)\right]^{\frac{1}{2}}} + \left[\frac{(297\alpha_N - 50)f_2(\alpha_N)\, Pr}{980\alpha_N^2}\right]^{\frac{3}{4}}\left[\frac{Gr_R}{Re_R^2}\right]^{\frac{3}{4}}\right\}^{\frac{1}{3}} \tag{42}$$

The results of the above analysis are borne out by Figures 2 and 3. It is interesting to note that the effect of viscoelasticity is to increase the Nusselt numbers and more so at higher Prandtl numbers. Moreover, in the stagnation region of a heated horizontal cylinder, natural convection constructively acts to increase the Nusselt number and thus ameliorates the heat transfer.

NOTATION

B_1, B_2, B_3 = constants in Equations (15), (16), (17)
C_p = specific heat per unit mass
$f_1(\alpha_F)$ = function of α_F defined by Equation (12)
$f_2(\alpha_N)$ = function of α_N defined by Equation (37)
g = acceleration due to gravity
Gr = Grashof number
Gr_x = local Grashof number defined in Equation (38)
Gr_R = Grashof number based on the radius of the cylinder defined in Equation (38)
I = integral defined by Equation (14)
k = thermal conductivity of the fluid
K = material constant
l_c = characteristic length
m = material constant
n = exponent in shear stress power-law
Nu_D = average Nusselt number for forced convection based on the diameter of the cylinder
Nu_M = average Nusselt number for mixed convection
$Nu_{x,N}$ = local Nusselt number for natural convection
$Nu_{x,F}$ = local Nusselt number for forced convection
$Nu_{avR,F}$ = average Nusselt number for forced convection based on the radius of the cylinder
$Nu_{avR,M}$ = average Nusselt number for mixed convection based on the radius of the cylinder
$Nu_{avR,N}$ = average Nusselt number for free convection based on the radius of the cylinder
p = exponent in Equation (17)
Pr = Prandtl number
r = exponent in Equation (15)
R = radius of the cylinder
Re = Reynolds number
Re_c = characteristic generalised Reynolds number defined by Equation (3)
Re_D = Reynolds number based on diameter of the cylinder
Re_x = local Reynolds number defined in Equation (33)
Re_R = Reynolds number based on the radius of the cylinder
s = exponent in normal stress power-law
t = exponent in Equation (16)
T = temperature
T_w = temperature of the solid surface
T_∞ = temperature of bulk of the fluid

u	= velocity component along x-co-ordinate
u_1	= dimensionless velocity component defined in Equation (3)
U	= velocity component outside the boundary layer
U_1	= dimensionless velocity component defined in Equation (3)
U_∞	= velocity of free stream
Wi	= Weissenberg number defined by Equations (4) and (39)
x	= distance along the curved surface
x_1	= dimensionless distance defined in Equation (3)
y	= distance normal to the curved surface
y_1	= dimensionless distance defined in Equation (3)

Greek Letters

α_F	= ratio of the thermal boundary layer thickness to the momentum boundary layer thickness for pure forced convection
α_N	= ratio of the thermal boundary layer thickness to the momentum boundary layer thickness for pure free convection
β	= expansion coefficient of the fluid
δ	= momentum boundary layer thickness
δ_1	= dimensionless momentum boundary layer thickness defined in Equation (3)
δ_T	= thermal boundary layer thickness
δ_{T_1}	= dimensionless thermal boundary layer thickness defined in Equation (3)
η	= similarity variable defined in Equation (9)
η_T	= similarity variable defined in Equation (9)
θ	= dimensionless temperature difference defined in Equation (3)
μ	= viscosity of the second order fluid
ρ	= density of the fluid

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Exact Solution of a Model for Diffusion in Particles and Longitudinal Dispersion in Packed Beds

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The problem of mass and heat transfer during flow through a packed bed has numerous applications in the chemical process industries. Theoretical studies of longitudinal dispersion, of either thermal energy or component concentration, in fixed-bed systems are readily available in the literature, and the analogy between heat conduction and component diffusion renders the analyses interchangeable. The following set of equations, which go back to the work by Deisler and Wilhelm (1953), has been employed by several authors

$$\frac{\partial C}{\partial t} + V\frac{\partial C}{\partial z} - D_L\frac{\partial^2 C}{\partial z^2} = -\frac{1}{m}\left(\frac{\partial \hat{q}}{\partial t}\right) \tag{1}$$

$$\frac{\partial q_i}{\partial t} = D_s\left(\frac{\partial^2 q_i}{\partial r^2} + \frac{2}{r}\frac{\partial q_i}{\partial r}\right) \tag{2}$$

The terms in the first equation stand for accumulation in the fluid phase, convective transport, transport by axial dispersion and volume-averaged accumulation in the spherical porous particles. In the second equation, the terms give accumulation in the particles and radial diffusion respectively. The assumptions leading to these equations have been discussed by Babcock et al. (1966) and by Pellett (1966). The boundary conditions commonly used are

$$C(0, t) = C_o \tag{3}$$

$$C(\infty, t) = 0 \tag{4}$$

$$C(z, 0) = 0 \tag{5}$$

$$q_i(0, z, t) \neq \infty \tag{6}$$

$$q_i(b, z, t) = q_s(z, t) \text{ given by } \frac{\partial \hat{q}}{\partial t} = \frac{3k_f}{b}\left(C - \frac{q_s}{K}\right) \tag{7}$$

$$q_i(r, z, 0) = 0 \tag{8}$$

The boundary condition (7) is the link between Equations (1) and (2). It states mathematically that the mass entering or leaving the particles must equal the flow of mass transported across a stagnant fluid film at the external surface.

For the case with no dispersion ($D_L = 0$), a classical solution of Equations (1) and (2) subject to the boundary conditions (3)-(8) was given by Rosen (1952) in terms of an infinite integral. Babcock et al. (1966) and Pellett (1966) have presented analytical solutions for the case including dispersion. Approximate solutions have been given by Radeke et al. (1976).

The solution of Babcock et al. (1966) was found to be in error for $D_L > 0$. For short times, values of C/C_o are predicted that are independent of time (Figure 1). It is to be shown below that Babcock's solution is actually a limiting solution for low values of

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